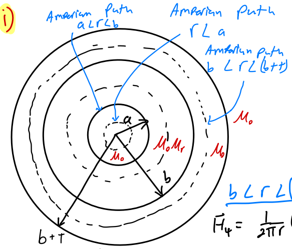
Infinitly Long Cable $-$ a, b, t, $\mu = \mu_0 \mu_r$

$\underbrace{a, b, t}_{\text{thickness}}$

i)



ii) field intensity

$r < a$:

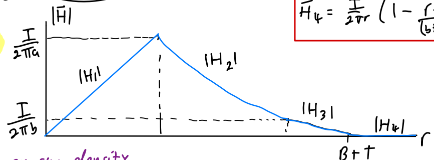
$$\oint H \cdot dl = I'$$
$$H\phi 2\pi r = \frac{I}{\pi a^2}\pi r^2$$
$$H\phi = \frac{Ir}{2\pi a^2}$$
$$\boxed{\vec{H}_1 = \frac{Ir}{2\pi a^2}\hat{\phi}}$$

$a < r < b$:
$$\boxed{\vec{H}_2 = \frac{I}{2\pi r}\hat{\phi}}$$

$r > (b+t)$:
$$\boxed{\vec{H}_3 = \frac{1}{2\pi r}(I - I) = 0}$$

$b < r < (b+t)$:
$$\vec{H}_4 = \frac{1}{2\pi r}(I - I')\hat{\phi} = \frac{1}{2\pi r}\left[I - \frac{I(\pi r^2 - \pi b^2)}{\pi(b+t) - \pi b^2}\right]\hat{\phi}$$
$$\boxed{\vec{H}_4 = \frac{I}{2\pi r}\left(1 - \frac{r^2 - b^2}{(b+t)^2 - b^2}\right)\hat{\phi}}$$

iii)



iv) energy density

$$w_{h_1} = \frac{\mu}{2}|H_1|^2 = \frac{\mu_0}{2}\frac{I^2 r^2}{4\pi^2 a^2} = \frac{\mu_0 I^2 r^2}{8\pi^2 a^2} \qquad 0 < r < a$$
$$w_{h_2} = \frac{\mu}{2}|H_2|^2 = \frac{\mu_0\mu_r}{2}\frac{I^2}{4\pi^2 r^2} = \frac{\mu_0\mu_r I^2}{8\pi^2 r^2} \qquad a < r < b$$

v) Total work done

$$W_{H_1} = \int_V w_{h_1}\, dv = \frac{\mu_0 I^2}{8\pi^2 a^2}\cdot\int_V r^2 \cdot r\, dr\, d\phi\, dz = \frac{\mu_0 I^2}{8\pi^2 a^2}\int_0^a r^3 dr \int_0^{2\pi} d\phi \int_0^1 dz$$
$$\boxed{W_{H_1} = \frac{\mu_0 I^2}{16\pi}}$$
$$W_{H_2} = \int_V w_{h_2} dv = \frac{\mu_0\mu_r I^2}{8\pi^2}\int_a^b \frac{1}{r^2}\cdot r\, dr \int_0^{2\pi} d\phi \int_0^1 dz$$
$$\boxed{W_{H_2} = \frac{\mu_0\mu_r I^2}{4\pi}\ln\left(\frac{b}{a}\right)}$$
$$\boxed{W_H = W_{H_1} + W_{H_2} = \frac{\mu_0 I^2}{4\pi}\left[\frac{1}{4} + \mu_r \ln\left(\frac{b}{a}\right)\right]}$$

vi)
$$W_H = \frac{1}{2}LI^2$$
$$\boxed{L = \frac{2W_H}{I^2} = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \mu_r\ln\left(\frac{b}{a}\right)\right]}$$

Laplace - Dielectric

2 concentric cylinders, a, b

$0V$ at $r = a$, $100V$ at $r = b$

General stuff:

$\nabla^2 V = 0 \rightarrow$ Laplace equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0$$
$$\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0$$

Integrate once

$$r\frac{\partial V}{\partial r} = A$$
$$\frac{\partial V}{\partial r} = \frac{A}{r}$$

Integrate again

$$V = A\ln(r) + B$$

at $r = a$, $V = 0$:
$$0 = A\ln(a) + B$$
at $r = b$, $V = 100$:
$$100 = A\ln(b) + B$$

$$\therefore A = \frac{100}{\ln\left(\frac{b}{a}\right)}$$
$$B = -\frac{100}{\ln\left(\frac{b}{a}\right)}\ln(a)$$

a) Complete Solution for V in dielectric

$$V(r) = \frac{100}{\ln\left(\frac{b}{a}\right)}\ln(r) - \frac{100}{\ln\left(\frac{b}{a}\right)}\ln(a) = \boxed{\frac{100}{\ln\left(\frac{b}{a}\right)}\ln\left(\frac{r}{a}\right)}$$

B) electric field intensity

$$\vec{E} = -\nabla V = -\frac{\partial V}{\partial r}\hat{r} = -\frac{A}{r}\hat{r} = \boxed{\frac{-100}{r\cdot\ln\left(\frac{b}{a}\right)}\hat{r}}$$

c) charge density

$$\rho_s = |\vec{D}| = \varepsilon|\vec{E}| = \boxed{\frac{-400\,\varepsilon_0}{r\cdot\ln(b/a)}}$$

Negative sign indicates lower potential than $r = b$

D)
$$C = \frac{|Q|}{V} = \frac{8\pi\varepsilon_0}{\ln(b/a)}$$
$$|Q| = \left|\int_S \rho_s\, ds\right| = \left|\frac{-400\,\varepsilon_0}{a\ln(b/a)}2\pi a\right| = \boxed{\frac{800\pi\varepsilon_0}{\ln\left(\frac{b}{a}\right)}}$$

e) leakage resistance

$$RC = \frac{\varepsilon}{\sigma} \qquad \therefore R = \frac{4\varepsilon_0}{\sigma\frac{8\pi\varepsilon_0}{\ln\left(\frac{b}{a}\right)}} = \boxed{\frac{\ln\left(\frac{b}{a}\right)}{2\pi\sigma}}$$

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Generic Formulas

$$\int_V dV \Rightarrow R^2\sin\theta\, dr\, d\theta\, d\phi$$
$$\int_S dA \Rightarrow R^2\sin\theta\, d\theta\, d\phi$$
$$\int_L dl = R\, d\phi$$

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$$
$$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$$

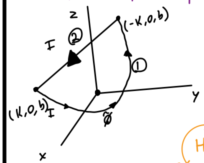
$\vec{r}$ = point of observation

$\vec{r}'$ = generic point on a source

Biot - Savart

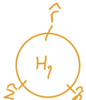
$$\vec{H} = \int_S \frac{I\,d\vec{l}' \times (\vec{r} - \vec{r}')}{4\pi|\vec{r} - \vec{r}'|^3}$$

Semi circle filament at $z = b$, Current I flows from $(K, 0, b)$ in ϕ direction to $(-K, 0, b)$, Then straight back to $(K, 0, b)$



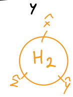
Segment 1

$$\vec{r} = 0$$
$$\vec{r}' = K\hat{r} + b\hat{z}$$
$$\vec{r} - \vec{r}' = -K\hat{r} - b\hat{z}$$
$$|\vec{r} - \vec{r}'| = \sqrt{K^2 + b^2}$$
$$d\vec{l} = K\,d\phi\,\hat{\phi}$$



$$\vec{H}_1 = \int \frac{IK\,d\phi\,\hat{\phi} \times (-K\hat{r} - b\hat{z})}{4\pi(K^2 + b^2)^{3/2}}$$
$$= \underbrace{\frac{IK}{4\pi(K^2+b^2)^{3/2}}}_{C}\left[\int_0^\pi K\,d\phi\,\hat{z} - b\int_0^\pi d\phi\,\hat{r}\right]$$
$$= C\left[K\pi\hat{z} - b\int_0^\pi \cos\phi\, d\phi\,\hat{x} + \sin\phi\, d\phi\,\hat{y}\right]$$
$$= C\left[K\pi\hat{z} - b\left[\sin\phi\right]_0^\pi\hat{x} + b\left[\cos\phi\right]_0^\pi\hat{y}\right]$$
$$\boxed{\vec{H}_1 = \frac{IK}{4\pi(K^2+b^2)^{3/2}}\left[K\pi\hat{z} - 2b\hat{y}\right]}$$

Segment 2



$$\vec{r} = 0$$
$$\vec{r}' = x\hat{x} + b\hat{z}$$
$$\vec{r} - \vec{r}' = -x\hat{x} - b\hat{z}$$
$$|\vec{r} - \vec{r}'| = \sqrt{x^2 + b^2}$$
$$d\vec{l} = dx\,\hat{x}$$

$$\vec{H}_2 = \int_{-K}^{K}\frac{I\,dx\,\hat{x} \times (-x\hat{x} - b\hat{z})}{4\pi(x^2 + b^2)^{3/2}}$$
$$= \frac{I}{4\pi}\int_{-K}^{K}\frac{b\,dx\,\hat{y}}{(x^2+b^2)^{3/2}}$$
$$= \frac{Ib}{4\pi}\hat{y}\left[\int_{-K}^{K}\frac{dx}{(x^2+b^2)^{3/2}}\right] = \left[\frac{x/b^2}{\sqrt{x^2+b^2}}\right]_{-K}^{K} = \frac{2K}{b^2\sqrt{x^2+b^2}}$$
$$\vec{H}_2 = \frac{I\,b\cdot 2K}{4\pi\,b^2\sqrt{x^2+b^2}}\hat{y}$$
$$\boxed{\vec{H}_2 = \frac{IK}{2\pi b\sqrt{x^2+b^2}}\hat{y}}$$

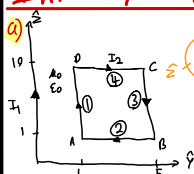
$$\vec{H} = \vec{H}_1 + \vec{H}_2$$
$$\boxed{\vec{H} = \frac{IK}{2\pi\sqrt{K^2+b^2}}\left[\frac{1}{b}\hat{y} + \frac{K\pi}{2(K^2+b^2)}\hat{z} - \frac{b}{(K^2+b^2)}\hat{y}\right]}$$

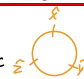
Infinitly Long Filament

I_1 in $\hat{z}$

I_2 in clockwise

a)





b) $\boxed{\vec{B} = \vec{B}_1 = \frac{\mu_0 I_1}{2\pi r}\phi}$

c)
$$\vec{F}_1 = \int I_2\, d\vec{l}_2 \times \vec{B}_1$$
$$\vec{F}_1 = \int_1^{10}(I_2\, dz\,\hat{z}) \times \left(\frac{\mu_0 I_1}{2\pi}\hat{x}\right)$$
$$= -\frac{\mu_0 I_1 I_2}{2\pi}(9)\hat{y}$$
$$\vec{F}_3 = \int_{10}^{1} I_2\, d\vec{l}_2 \times \vec{B}_2$$
$$= \frac{\mu_0 I_1 I_2}{10\pi}(9)\hat{y}$$

$$\vec{F} = \vec{F}_1 + \cancel{\vec{F}_2} + \vec{F}_3 + \cancel{\vec{F}_4}$$
$$\hookrightarrow F_2 = -F_4$$
$$\therefore \vec{F} = \vec{F}_1 + \vec{F}_3$$
$$= -\frac{9\mu_0 I_1 I_2}{2\pi}\hat{y} + \frac{9\mu_0 I_1 I_2}{10\pi}\hat{y}$$
$$\vec{F} = -\frac{3.6\,\mu_0 I_1 I_2}{\pi}\hat{y}\ [N]$$

D) Force on filament due to Loop:
$$F_N = -F$$
$$F_N = \frac{3.6}{\pi}\mu_0 I_1 I_2\,\hat{y}\ [N]$$

Generic Questions

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Maxwell Equations

Gauss' law $\nabla\cdot\vec E = \frac{\rho}{\varepsilon_0}$	$\nabla\cdot\vec D = \rho_v$	$\oint_s \vec D\cdot d\vec s = Q$
Maxwell-Faraday $\nabla\times\vec E = -\frac{\partial B}{\partial t}$	$\nabla\times\vec E = -\frac{\partial B}{\partial t}$	$\oint_c \vec E\cdot dl = -\int_s \frac{\partial \vec B}{\partial t}\cdot d\vec s$
Gauss' Law for Magnetism $\nabla\cdot\vec B = 0$	$\nabla\cdot\vec B = 0$	$\oint_s \vec B\cdot d\vec s = 0$
Ampere Circuit $\nabla\times\vec B = \mu_0(\vec J + \varepsilon_0\frac{\partial E}{\partial t})$	$\nabla\times\vec H = J + \frac{\partial D}{\partial t}$	$\oint \vec H\cdot d\vec l = \int_s(\vec J + \frac{\partial D}{\partial t})ds$

Continuity of current

Point form: $\nabla\cdot\vec J = -\frac{\partial \rho_v}{\partial t}$ Integral form: $I_D = \int_s \frac{\partial \vec D}{\partial t}\cdot ds$

Displacement Current density

$\nabla\times\vec H = \vec J + \frac{\partial D}{\partial t} \Rightarrow \int_s(\nabla\times\vec H)\cdot d\vec s = \int_s \vec J\cdot d\vec s + \int_s \frac{\partial D}{\partial t}\cdot ds$

Stokes: $\oint_c \vec H\cdot d\vec l = I_c + \int_s \frac{\partial \vec D}{\partial t}\cdot ds$

$\therefore I_D = \int_s \vec J_D\cdot ds = \int_s \frac{\partial D}{\partial t}\cdot ds$ $\quad \therefore I_D = \frac{\partial D}{\partial t}$

Mechanisms that cause EMI with 3 fixes

Mechanism:
- ground
- Antenna effect
- cross talk
- switching operations
- Differential Mode

Fixes:
- shielding
- filtering and grounding
- Layout and component placement

Magnetic Boundry

$\mu_1 = \mu_0$, $\mu_2 = 500\mu_0$

Normal direction $\hat n = \hat r$

→ Normal/tangential component of magnetic flux

a) $B_{2n} = \vec B_2\cdot\hat n = 15$

$$\boxed{\vec B_{2n} = 15\hat r \qquad \vec B_{2t} = 100\hat\phi + 250\cos\phi\,\hat z}$$

b) → find B_1 with $\vec B_2$ at $\phi = 0°$ $\quad \vec B_2 = 15\hat r + 100\hat\phi + 250\hat z$

$\hat n\times(\vec H_1 - \vec H_2) = \vec J_s \qquad \vec H_1\times\hat r = \vec J_s + \vec H_2\times\hat r$

$= -J_0\hat z + \frac{\vec B_2}{\mu_2}$

$= -J_0\hat z + \frac{15\hat r + 100\hat\phi + 250\hat z}{\mu_2}\times\hat r$

$(H_{1r}\hat r + H_{1\phi}\hat\phi + H_{1z}\hat z)\times\hat r = -J_0\hat z - \frac{100\hat z}{\mu_2} + \frac{250\hat\phi}{\mu_2}$

$-H_{1\phi}\hat z + H_{1z}\hat\phi = -J_0\hat z - \frac{100\hat z}{\mu_2} + \frac{250\hat\phi}{\mu_2}$

$H_{1\phi} = J_0 + \frac{100}{\mu_2} \qquad H_{1z} = \frac{250}{\mu_2}$

$\vec H_{1t} = (J_0 + \frac{100}{\mu_2})\hat\phi + \frac{250}{\mu_2}\hat z$

$\vec B_t = \mu_1 H_{1t} = (\mu_0 J_0 + \frac{100}{500})\hat\phi + \frac{250}{500}\hat z$

$= (\mu_0 J_0 + 0.2)\hat\phi + 0.5\hat z$

$$\boxed{\vec B_1 = 15\hat r + (\mu_0 J_0 + 0.2)\hat\phi + 0.5\hat z}$$

c) → find B_1 with $\vec B_2$ at $\phi = 90°$ $\quad \vec B_2 = 15\hat r + 100\hat\phi$

$\vec B_{2n} = 15\hat r = \vec B_{1n} \qquad H_{2t} = H_{1t} \Rightarrow \frac{B_{2t}}{\mu_2} = \frac{B_{1t}}{\mu_1}$

$\vec B_{2t} = 100\hat\phi$

$\vec B_{1t} = \frac{\mu_1}{\mu_2}\vec B_{2t} = \frac{\mu_0}{500\mu_0}100\hat\phi = 0.2\hat\phi$

$$\boxed{\vec B_1 = 15\hat r + 0.2\hat\phi \text{ at } \hat\phi = 0°}$$

Stationary Loop

x-y plane, $a = 0.5$ m, $R_i = 0.4\,\Omega$, $R = 400\,\Omega$, $\omega = 10^6\,\frac{rad}{s}$

$\vec B(t) = 6(3\hat r - 5r\hat z)\cos\omega(t)$ Wb/m²

a) $\phi = \int \vec B(t)\cdot d\vec s$

$= \int_{r=0}^{a}\int_{\phi=0}^{2\pi}(18\hat r - 30r\hat z)\cos\omega t\cdot r\,d\theta\,dr\cdot\hat z$

$= +30\cos(\omega t)\int_0^a r^2dr\int_0^{2\pi}d\phi$

$= \frac{+60\pi(0.5)^3}{3}\cos(10)^6(t)$

$$\boxed{\phi = \frac{5\pi}{2}\cos(10^6 t)}$$

b) $V_{emf}^{tr} = -\int \frac{\partial \vec B}{\partial t}\cdot ds$

$= -\int_s \frac{\partial}{\partial t}(18\hat r - 30r\hat z)\cos\omega t\cdot r\,dr\,d\phi\,(\hat z)$

$= +\int_s \omega\sin\omega t(18\hat r - 30r\hat z)\cdot r\,dr\,d\phi\,(\hat z)$

$= \omega\sin\omega(t)\cdot 30\int_0^{0.5} r^2 dr\int_0^{2\pi} d\phi$

$= \frac{30\times(0.5)^3}{3}\times 2\pi\ \omega\sin(\omega t) = 2.5\sin\omega t$

d) Set $\omega = 10^6$ and $\omega t = \frac{5\pi}{4}$

at $\omega t = \frac{5\pi}{4}$ $\quad V_{emf}^{tr} = -1.76\times10^6$ V

$I = \frac{V_{emf}^{tr}}{R} = \frac{V_1 - V_2}{400} = \frac{-1.76\times10^8}{400}$

$I = -4.42$ in counterclockwise

Moving Loop in Static field

$u = -10\hat y$ m/s

$B(y) = 5\cos(0.1y)\hat z$ Wb/m²

a) B at $y_1 \Rightarrow B(y_1) = 5\cos(0.1\times4)\hat z$

$\therefore \boxed{B(y_1) = 4.605 \text{ Wb/m}^2}$

b) $V_{12} = V_{emf}^m = \int_l \vec u\times\vec B\cdot d\vec l$

$= \int_1^{-1}(-10\hat y\times 4.605\hat z)\cdot dl\,\hat x$

$$\boxed{V_{12} = 92.1 \text{ V}}$$

c) B at $y_2 \Rightarrow B(y_2) = 5\cos(0.1\times4.5)\hat z$

$$\boxed{B(y_2) = 4.502\hat z \text{ Wb/m}^2}$$

d) $V_{43} = \int(-10\hat y\times 4.502\hat z)\,dl\,\hat x$

$$\boxed{V_{43} = 90.04 \text{ V}}$$

e) $V_{41} = V_{32} = 0$

f) $I = \frac{V_{12} - V_{43}}{R} = \frac{92.1 - 90.04}{R} = \frac{1.7}{25}$ counter clockwise

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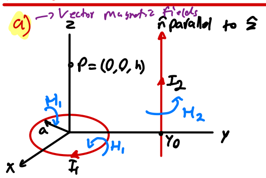
Magnetic field of a wire

a) → Vector magnetic fields



b) → Find Magnetic field H

for I_2:

$$\vec{H}_2 = \frac{I_2}{2\pi y_0}\hat{\phi}$$

$$= \frac{I_2}{2\pi y_0}\left[-\sin\phi\,\hat{x} + \cos\phi\,\hat{y}\right] \quad \phi = -\frac{\pi}{2}$$

$$\boxed{\vec{H}_2 = \frac{I_2}{2\pi y_0}\hat{x}}$$

c) → find H given y_0, a, h, I_1, I_2

$$\boxed{H = -\frac{20 \times 5^2}{2(5^2+6^2)^{3/2}} + \frac{30}{2\pi(20)} \ [A/cm]}$$

$\vec{r} = h\hat{z}$

$\vec{r}\,' = a\hat{r}$

$\vec{r} - \vec{r}\,' = h\hat{z} - a\hat{r}$

$|\vec{r} - \vec{r}\,'| = \sqrt{h^2+a^2}$

$I\,dL = I_1\, a\, d\phi$

for I_1:

$$\vec{H}_1 = \int \frac{I\,d_l \times (\vec{r} - \vec{r}\,')}{4\pi|\vec{r} - \vec{r}\,'|^2}$$

$$= \frac{-I_1 a}{4\pi(a^2+h^2)^{3/2}}\left[\int_0^{2\pi} h\,d\phi\,\hat{r} + \int_0^{2\pi} a\,d\phi\,\hat{z}\right]$$

$$= \frac{-I_1 a}{4\pi(a^2+h^2)^{3/2}}\left(a(2\pi)\hat{z}\right)$$

$$\boxed{\vec{H}_1 = \frac{-I_1 a^2}{2(a^2+h^2)^{3/2}}\hat{z}}$$

$$\boxed{\vec{H} = \vec{H}_1 + \vec{H}_2}$$

d) $\vec{F} = I\,\vec{l} \times \vec{B}$

$\vec{F} = I \oint_c dl \times \vec{B}$

Amperes law for cylinder

a) $\oint \vec{H}\cdot d\vec{l} = \int \vec{J}\cdot d\vec{s}$

$$\Rightarrow H_\phi 2\pi r = \int \frac{J_0}{r} e^{-r}\hat{z} \cdot r\,dr\,d\phi\,\hat{z}$$

$$H_\phi 2\pi r = J_0 \int_0^r e^{-r}\,dr \int_0^{2\pi} d\phi$$

$$H_\phi r = -J_0\left(e^{-r} - 1\right)$$

$$\boxed{\vec{H}_1 = -\frac{J_0}{r}\left(1 - e^{-r}\right)\phi}$$

b) $\boxed{H_2 = \frac{J_0}{r}\left(1 - e^{-a}\right)\phi}$

c) $|H_1| = \frac{3}{5}\left(1 - e^{-5}\right)$

$|H_2|\ \frac{3}{5}\left(1 - e^{-5}\right)$

Same

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